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WEAK RECONSTRUCTION OF EDGE-DELETED CARTESIAN PRODUCTS

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In 1960 Ulam [4] asked whether a graph G is uniquely determined up to isomorphisms by its deck, that is, by the set of all graphs $G \setminus x$ obtained from G by deleting a vertex x and all edges incident to it. This led to the *Reconstruction Conjecture*, also known as *Ulam's Conjecture*, that any two graphs on at least three vertices with the same deck are isomorphic. This still is open for finite graphs. When reconstructing a class of graphs, the problem partitions into the subproblems *recognition* and *weak reconstruction*. The first consists of showing that membership in the class is determined by the deck, and the latter that nonisomorphic members of the class have different decks.

Imrich and Žerovnik [3], who showed that both the recognition and the weak reconstruction problem can be solved from a single vertex-deleted subgraph for nontrivial, connected finite or infinite Cartesian products.

In 1964 Harary [2] introduced the *Edge Reconstruction Conjecture*, that any two graphs with at least four edges that have the same deck of edge-deleted subgraphs are isomorphic. For products this was taken up by Dörfler [1], who showed that all nontrivial strong products and certain lexicographic products can be reconstructed from the deck of all edge-deleted subgraphs. He did not treat the edge-reconstruction of Cartesian products.

Here we show that both the *recognition problem* and the *weak edge reconstruction problem* can be solved from a single edge-deleted subgraph for nontrivial, connected finite or infinite Cartesian products.

For finite graphs G the reconstruction is possible in $O(mn^2)$ time, where n is the order and m the size of G .

Literatura

- [1] W. Dörfler, On the edge-reconstruction of graphs, Bull. Austral. Math. Soc. 10 (1974), 79–84.
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- [4] S. M. Ulam, A Collection of Mathematical Problems, Wiley, New York, 1960, p.29.