



## SEMINARIUM MATEMATYKA DYSKRETNA

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### ON VARIATIONS OF THE 1-2-3 CONJECTURE

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The *1-2-3 Conjecture*, raised by Karoński, Łuczak and Thomason, states that every connected graph different from  $K_2$  admits a proper 3-labelling, i.e., can have its edges labelled with 1, 2 or 3 so that no two adjacent vertices have the same sum of labels. Following a recent trend, we introduce an optimisation variation of this conjecture, in which we study proper labellings whose sum of assigned labels is as small as possible. Our investigation leads us to conjecture that for almost every graph  $G$ , there should exist a proper labelling with label sum at most  $2|E(G)|$ .

The intuition behind this new conjecture we proposed, which also seems to be supported by previous investigations, is that, in general, it should always be possible to produce proper 3-labellings assigning label 3 to “only few edges”. We proceed by formally investigating proper labellings that minimise the number of edges labelled 3. As it turns out, there exist families of graphs that require an unbounded number of edges labelled 3. For these families, we give upper bounds on the number of edges labelled 3, leading us to conjecture that, for almost every graph  $G$ , there should exist a proper labelling that assigns label 3 to at most  $\frac{1}{3}|E(G)|$ .

Finally, it is worth mentioning the recent work of Baudon, Pilśniak, Przybyło, Senhaji, Sopena, and Woźniak, in which they study proper labellings that are *equitable*, i.e., every two labels are assigned about the same number of times. Their work makes us wonder about the plausibility of an *Equitable 1-2-3 Conjecture*, which, if it holds, would directly imply the two conjectures we proposed above.