# Single-conflict coloring and Flexibility: Degeneracy and Maximum degree bounds

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We will look at two coloring problems and we will examine two methods that give us results based on the degeneracy or maximum degree of a graph.

First, we consider single-conflict colorings, a variant of graph colorings in which each edge of a graph has a single forbidden color pair. We show that for any assignment of forbidden color pairs to the edges of a $d$-degenerate graph $G$ on $n$ vertices of edge-multiplicity at most $\log \log n$, $O(\sqrt{d} \log n)$ colors are always enough to color the vertices of $G$ in a way that avoids every forbidden color pair. This answers a question of Dvořák, Esperet, Kang, and Ozeki for simple graphs [JGT 2021].

Second, we consider the flexibility problem. For a given $\varepsilon > 0$, we say that a graph $G$ is $\varepsilon$-flexibly $k$-choosable if the following holds: for any assignment $L$ of color lists of size $k$ on $V(G)$, if a preferred color from a list is requested at any set $R$ of vertices, then at least $\varepsilon|R|$ of these requests are satisfied by some $L$-coloring. We consider the question of flexible choosability in several graph classes with certain degeneracy conditions. We characterize the graphs of maximum degree $\Delta$ that are $\varepsilon$-flexibly $\Delta$-choosable for some $\varepsilon = \varepsilon(\Delta) > 0$, which answers a question of Dvořák, Norin, and Postle [JGT 2019]. In particular, we show that for any $\Delta \geqslant 3$, any graph of maximum degree $\Delta$ that is not isomorphic to $K_{\Delta+1}$ is $1/(6\Delta)$-flexibly $\Delta$-choosable. Our fraction of $1/(6\Delta)$ is within a constant factor of being the best possible. We also show that graphs of treewidth 2 are $1/3$-flexibly $3$-choosable, answering a question of Choi, Clemen, Ferrara, Horn, Ma, and myself [DAM 2021].

We will also discuss some recent developments and open questions.

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